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Senior Secondary Australian Curriculum

Mathematical Methods Glossary

Unit 1

Functions and graphs

Asymptote

A line is an **asymptote** to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \pi/2$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = 1/x$.

Binomial distribution

The expansion $(x + y)^n = x^n + \binom{n}{1} x^{n-1}y + \cdots + \binom{n}{r} x^{n-r}y^r + \cdots + y^n$ is known as the **binomial theorem**. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\times(n-1)\times\cdots\times(n-r+1)}{r\times(r-1)\times\cdots\times2\times1}$ are called binomial coefficients.

Completing the square

The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Rewriting it in this way is called **completing the square**.

Discriminant

The **discriminant** of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$

Function

A **function** f is a rule that associates with each element x in a set S a unique element $f(x)$ in a set T. We write $x \longmapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the **domain** of f and the set T is called the **codomain**. The subset of T consisting of all the elements $f(x): x \in S$ is called the **range** of f. If we write $y = f(x)$ we say that x is the **independent variable** and y is the **dependent variable.**

Graph of a function

The **graph of a function** f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$

Quadratic formula

If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. This formula for the roots is called the **quadratic formula**.

Vertical line test

A relation between two real variables x and y is a function and $y = f(x)$ for some function f, if and only if each vertical line, i.e. each line parallel to the $y -$ axis, intersects the graph of the relation in at most one point. This test to determine whether a relation is, in fact, a function is known as the **vertical line test**.

Trigonometric functions

Circular measure is the measurement of angle size in radians.

Radian measure

The **radian measure** θ of an angle in a sector of a circle is defined by $\theta = \ell/r$, where r is the radius and ℓ is the arc length. Thus an angle whose degree measure is 180 has radian measure π.

Length of an arc

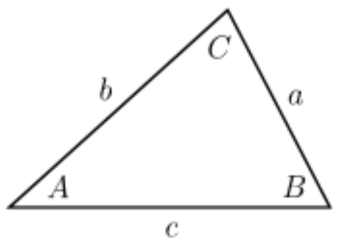
The **length of an arc in a circle** is given by $\ell = r\theta$, where ℓ is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

Sine rule and cosine rule

The lengths of the sides of a triangle are related to the sines of its angles by the equations

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

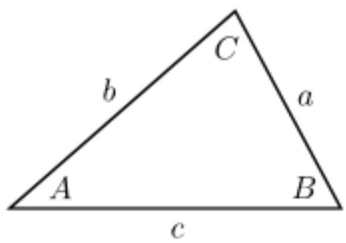
This is known as the **sine rule**.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

$$c^2 = a^2 + b^2 - 2ab\cos C$$

This is known as the **cosine rule**.



Sine and cosine functions

In the unit circle definition of cosine and sine, $\cos\theta$ and $\sin\theta$ are the x and y coordinates of the point on the unit circle corresponding to the angle θ

Period of a function

The period of a function $f(x)$ is the smallest positive number p with the property that $f(x+p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π

Counting and Probability

Pascal's triangle

Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the **binomial coefficients** $\binom{n}{r}$, for $0 \le r \le n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n

Conditional probability

The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a **conditional probability** and is written as $P(A|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called **the reduced sample space**, is B. The conditional probability of event A is given by $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

Independent events

Two events are **independent** if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A|B) = P(A)$ or if $P(B) = P(B|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.

Mutually exclusive

Two events are **mutually exclusive** if there is no outcome in which both events occur.

Point and interval estimates

In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probability or population parameter. If the estimate is a single number, this number is called a **point estimate.** An **interval estimate** is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.

Relative frequency

If an event E occurs r times when a chance experiment is repeated n times, the **relative** frequency of E is r/n.

Unit 2

Exponential functions

Index laws

The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers x, y, a and b, with $a > 0$ and $b > 0$

Algebraic properties of exponential functions

The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers x, y, and a, with $a > 0$

Arithmetic and Geometric sequences and series

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is *a* and the common difference of successive members is *d*, then the *n*th term t_n, of the sequence, is given by:

$t_n = a + (n - 1)d$ for $n \geq 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = t_n + d$ where *d* is the common difference and $n \geq 1$.

Geometric sequence

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the **common ratio**. For example, the sequence

3, 6, 12, 24, ...

is a geometric sequence with common ratio 2. Similarly the sequence

40, 20, 10, 5, 2.5, …

is a geometric sequence with common ratio $\frac{1}{2}$.

If the initial term of a geometric sequence is *a* and the common ratio of successive members is *r*, then the *n*th term t_n, of the sequence, is given by:

$t_n = ar^{n-1}$ for $n \geq 1$

A recursive definition is

$t_1 = a, \quad t_{n+1} = rt_n$ for $n \geq 1$ and where r is the constant ratio

Partial sums of a sequence (Series)

The sequence of partial sums of a sequence $t_1, \ldots, t_s, \ldots$ is defined by

$S_n = t_1 + \ldots + t_n$

Partial sum of an arithmetic sequence (Arithmetic series)

The partial sum S_n of the first n terms of an arithmetic sequence with first term a and common difference d.

$a, a + d, a + 2d, \ldots\ldots, a + (n - 1)d, \ldots$

is

$S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}(2a + (n - 1)d)$ where t_n is the n^{th} term of the sequence.

The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$

Partial sums of a geometric sequence (Geometric series)

The partial sum S_n of the first n terms of a geometric sequence with first term a and common ratio r,

$a, ar, ar^2, \ldots\ldots, ar^{n-1}, \ldots$

is

$S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1.$

The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$.

Introduction to differential calculus

Gradient (Slope)

The **gradient** of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2 - y_1}{x_2 - x_1}$.

Slope is a synonym for **gradient**.

Secant

A **secant** of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a **chord**.

Tangent line

The **tangent line** (or simply the **tangent**) to a curve at a given point P can be described intuitively as the straight line that "just touches" the curve at that point. At P the curve meet, the curve has "the same direction" as the tangent line. In this sense it is the best straight-line approximation to the curve at the point P.

Linearity property of the derivative

The **linearity property of the derivative** is summarized by the equations:

$\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k

and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

Local and global maximum and minimum

A **stationary point** on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$.

We say that $f(x_0)$ is a **local maximum** of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a **global maximum** of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x in the domain of f.

We say that $f(x_0)$ is a **local minimum** of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a **global minimum** of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f.

Unit 3

Further differentiation and applications

Euler's number

Euler's number e is an irrational number whose decimal expansion begins

$$e = 2.7182818284590452353602874713527 \cdots$$

It is the base of the natural logarithms, and can be defined in various ways including:

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$.

Product rule

The **product rule** relates the derivative of the product of two functions to the functions and their derivatives.

If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$,

and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quotient rule

The **quotient rule** relates the derivative of the quotient of two functions to the functions and their derivatives

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Composition of functions

If $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$

Chain rule

The **chain rule** relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)'(x) = f'\big(g(x)\big)g'(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Concave up and concave down

A graph of $y = f(x)$ is concave up at a point P if points on the graph near P lie above the tangent at P. The graph is concave down at P if points on the graph near P lie below the tangent at P.

Point of inflection

A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P

Second derivative test

According to the second derivative test, if $f'(x) = 0$, then $f(x)$ is a local maximum of f if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$

Integrals

Antidifferentiation

An **anti-derivative**, **primitive** or **indefinite integral** of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$.

The process of solving for anti-derivatives is called **anti-differentiation**.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\,dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the **constant of integration**. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$

The linearity property of anti-differentiation

The linearity property of anti-differentiation is summarized by the equations:

$\int kf(x)dx = k\int f(x)dx$ for any constant k and

$\int (f_1(x) + f_2(x))dx = \int f_1(x)dx + \int f_2(x)\,dx$ for any two functions $f_1(x)$ and $f_2(x)$

Similar equations describe the linearity property of definite integrals:

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for any constant k and

$\int_a^b (f_1(x) + f_2(x))dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$ for any two functions $f_1(x)$ and $f_2(x)$

Additivity property of definite integrals

The **additivity property of definite integrals** refers to 'addition of intervals of integration':

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

The fundamental theorem of calculus

The **fundamental theorem of calculus** relates differentiation and definite integrals. It has two forms:
$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ and $\int_a^b f'(x)dx = f(b) - f(a)$

Discrete random variables

Random variable

A **random variable** is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A **discrete random variable** is one whose possible values are the counting numbers $0,1,2,3,\cdots$, or form a finite set, as in the first two examples.

A **continuous random variable** is one whose set of possible values are all of the real numbers in some interval.

Probability distribution

The **probability distribution** of a discrete random variable is the set of probabilities for each of its possible values.

Uniform discrete random variable

A **uniform discrete random variable** is one whose possible values have equal probability of occurrence. If there are n possible values, the probability of occurrence of any one of them is $1/n$.

Expected value

The **expected value** $E(X)$ of a random variable X is a measure of the central tendency of its distribution.

If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and

$p_i = P(X = x_i)$.

If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X

Mean of a random variable

The **mean** of a random variable is another name for its expected value.

Variance of a random variable

The **variance** $Var(X)$ of a random variable X is a measure of the 'spread' of its distribution.

If X is discrete, $Var(X) = \sum_i p_i(x_i - \mu)^2$, where $\mu = E(X)$ is the expected value.

If X is continuous, $Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$

Standard deviation of a random variable

The **standard deviation** of a random variable is the square root of its variance.

Effect of linear change

The **effects of linear changes of scale and origin** on the mean and variance of a random variable are summarized as follows:

If X is a random variable and $Y = aX + b$, where a and b are constants, then

$E(Y) = aE(X) + b$ and $Var(Y) = a^2 Var(X)$

Bernoulli random variable

A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.

Bernoulli trial

A **Bernoulli trial** is a chance experiment with possible outcomes, typically labeled 'success' and failure'.

Unit 4

The logarithmic function

Algebraic properties of logarithms

The algebraic properties of logarithms are the rules: $\log_a(xy) = \log_a x + \log_a y$, $\log_a \frac{1}{x} = -\log_a x$, and $\log_a 1 = 0$, for any positive real numbers x, y and a

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Continuous random variables and the normal distribution

Probability density function

The **probability density function** of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\,dx$.

Uniform continuous random variable

A **uniform continuous random variable** X is one whose probability density function $p(x)$ has constant value on the range of possible values of X. If the range of possible values is the interval $[a, b]$ then $p(x) = \frac{1}{b-a}$ if $a \leq x \leq b$ and $p(x) = 0$ otherwise.

Triangular continuous random variable

A **triangular continuous random variable** X is one whose probability density function $p(x)$ has a graph with the shape of a triangle.

Quantile

A **quantile** t_α for a continuous random variable X is defined by $P(X > t_\alpha) = \alpha$, where $0 < \alpha < 1$.

The **median** m of X is the quantile corresponding to $\alpha = 0.5$: $P(X > m) = 0.5$

Interval estimates for proportions

Central limit theorem

There are various forms of the **Central limit theorem,** a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

"If $\bar{X}$ is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution."

In the special case where X is a Bernoulli random variable with parameter p, $\bar{X}$ is the sample proportion $\hat{p}$, $\mu = p$ and $\sigma = \sqrt{p(1-p)}$. In this case the Central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.

Margin of error

The **margin of error** of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.

Level of confidence

The **level of confidence** associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.